

# Geometric Evolution Equations and Related Fields

*Chih-Wei Chen (National Sun-Yat Sen University, Taiwan)*

## W-functionals in the Ricci flow and the Yamabe flow

### Abstract:

We use Perelman's W-functional to show that every closed Ricci flow has a volume lower bound which is determined by the initial data and is sharp on the standard sphere. This is a joint work with Zhenlei Zhang.

On the other hand, we construct a W-functional for Yamabe flow which can be used to study finite-time singularities. This work is still in progress, jointly with Shu-Cheng Chang and Kuo-Wei Lee.

*Qing-Ming Cheng (Fukuoka University, Japan)*

## Compact minimal hypersurfaces in $S^5(1)$

### Abstract:

In this talk, we consider  $n$ -dimensional minimal hypersurfaces with constant scalar curvature in the unit sphere. We will discuss Chern problems on compact minimal hypersurfaces with constant scalar curvature in the unit sphere. A survey of the recent progress on Chern problems is given. In particular, we mainly study 4-dimensional compact minimal hypersurfaces with constant scalar curvature in the unit spheres  $S^5(1)$ .

*Jui-En Chang (National Taiwan University, Taiwan)*

## Generic singularities of the network flow

### Abstract:

In the network flow, singularities may form. They can be described as self-similar shrinking solutions called regular shrinkers. An important problem is that if we perturb the initial network, will the new network flow to the same singularity? All network with 2 or more enclosed regions can be perturbed away. Therefore, the problem reduces to the network with less than 2 enclosed regions. There are finitely many of them and they are completely classified. Here, I use the entropy argument as in Colding and Minicozzi's work to show that the 4-ray star, the 5-ray star, the fish, and the rocket can be perturbed away.

*Siao-Hau Guo (National Taiwan University, Taiwan)*

## Mean Curvature Flow and Self-Similar Solution

### Abstract:

The study of mean curvature flows (MCF) has led to many fascinating results and applications. In this talk, we will explore some interesting properties of MCF including its connection with the entropy and how self-similar solutions model the asymptotic behavior under certain conditions.

*Toru Kajigaya (Tokyo Denki University, Japan)*

## **Equivariant realizations of Hermitian symmetric space of noncompact type**

### **Abstract:**

Let  $M=G/K$  be a Hermitian symmetric space of noncompact type. We provide a way of constructing  $K$ -equivariant embeddings from  $M$  to its tangent space at the origin by using the polarity of the  $K$ -action. As an application, we reconstruct the  $K$ -equivariant holomorphic embedding so called the Harish-Chandra realization and the  $K$ -equivariant symplectomorphism constructed by Di Scala-Loi and Roos under appropriate identifications of spaces. Moreover, we characterize the holomorphic/symplectic embedding of  $M$  by means of the polarity of the  $K$ -action. Furthermore, we discuss realizations of totally geodesic submanifolds by the  $K$ -equivariant embeddings and a dual map on the compact dual of  $M$ . This talk is based on a joint work with Takahiro Hashinaga.

*Naoyuki Koike (Tokyo University of Science, Japan)*

## **The existence and the uniqueness of regularized mean curvature flows**

### **Abstract:**

The existence and the uniqueness of a mean curvature flow starting from a compact submanifold (without boundary) in a complete Riemannian manifolds in short time is shown by using de Turck trick. In more general, B. L. Chen and L. Yin ([CY]) showed the existence and the uniqueness of a mean curvature flow of bounded second fundamental form starting from a complete submanifold of bounded second fundamental form in a complete Riemannian manifold by using the de Turck trick, where we note that “the boundedness of second fundamental form” of the flow controls the behaviour at the infinity of the flow. In this talk, we state the similar results for the existence and the uniqueness of  $\mathcal{G}$ -invariant regularized mean curvature flows in a Hilbert space  $V$  equipped with a certain kind of Hilbert Lie group isometric action  $\mathcal{G} \curvearrowright V$ . Also, we state the existence of a regularized mean curvature flow of bounded shape operators starting from an isoparametric submanifold in a Hilbert space and the collapsing theorem along the flow. Furthermore, we state a plan to apply the study of a regularized mean curvature flow to the gauge theory.

### REFERENCES

- [CY] B. L. Chen and L. Yin, Uniqueness and pseudolocality theorems of the mean curvature flow, *Comm. Anal. & Geometry* **15** (2007), 435–490.

*Keita Kunikawa (Utsunomiya University, Japan)*

## **Liouville theorem for heat equation along ancient Ricci flow**

### **Abstract:**

In this talk, I will show a Liouville type theorem for heat equation along ancient Ricci flow. We formulate such a Liouville theorem under a growth condition concerning Perelman’s reduced distance.

*Chun-Chi Lin (National Taiwan Normal University, Taiwan)*

## Elastic flows of networks in Euclidean spaces

### Abstract:

In this talk, I would like to present the  $L^2$ -gradient flow of the penalized elastic energy on networks of  $q$ -curves in  $\mathbb{R}^n$  for  $q \geq 3$ . Each curve is fixed at one end-point and at the other is joint to the other curves at a movable  $q$ -junction. For this geometric evolution problem with natural boundary conditions, we first show the existence of smooth solutions in short-time. Since the geometric problem is not well-posed, due to the freedom in reparametrization of curves, we consider a fourth-order non-degenerate parabolic quasilinear system, called the analytic problem, and show a short-time existence result for this parabolic system. The proof relies on applying Solonnikov's theory on linear parabolic systems and Banach fixed point theorem in proper Hölder spaces. Then the original geometric problem is solved by establishing the relation between the analytical solutions and the solutions to the geometrical problem. We then provide a long-time existence and sub-convergence result under some mild topological assumptions. The evolution is such that the sum of the elastic energies of the networks plus their weighted lengths decrease in time. To show the long-time existence, we obtain uniform bounds for  $L^2$ -norms of curvature derivatives of any order. This is a joint work with Anna Dall'Acqua (Uni. Ulm) and Paola Pozzi (Uni. Duisburg-Essen).

*Yukihiro Seki (OCAMI, Japan)*

## Description of non-self-similar singularities in harmonic map heat flow

### Abstract:

In this talk we will discuss singularity formation arising in the harmonic map heat flow from the  $d$ -dimensional Euclidean space to a unit sphere  $S^d$ . It is well known that classical Eells–Sampson theory fails in this case due to the occurrence of finite-time singularity for energy density. I will introduce typical examples of solutions which exhibit non-self-similar singularity. This talk is based on joint works with P. Biernat.

*Wei-Bo Su (Academia Sinica, Taiwan)*

## Construction of Lagrangian Translating Solitons

### Abstract:

Lagrangian translating solitons are Lagrangian submanifolds in Euclidean spaces which evolve by translations under mean curvature flow. They provide possible local models for Type II singularities of mean curvature flow. In this talk, I will describe a construction of new examples of Lagrangian translating solitons by desingularizing the intersection points between suitably rotated Grim Reaper cylinders using special Lagrangian Lawlor necks.

*Dr. Ryosuke Takahashi (Kyushu University, Japan)*

## **Some geometric flow approaches for deformed Hermitian Yang-Mills equation**

### **Abstract:**

On SYZ mirror symmetry, a deformed Hermitian Yang-Mills (dHYM) metric is a fiber metric on a holomorphic line bundle, which is the mirror object to a special Lagrangian section of the dual torus fibration. As a parabolic analogue, Jacob and Yau Yau introduced the Line Bundle Mean Curvature Flow (LBMCF) as the mirror of the Lagrangian Mean Curvature Flow. In this talk, we explore some geometric flow approaches for dHYM metrics as follows: (A) On Kähler surfaces, it is known that the existence of dHYM metrics is equivalent to a Kähler condition for a certain cohomology class. We relax this condition and study how the LBMCF blows up. (B) Recently, Collins and Yau Yau discovered a new variational characterization for dHYM metrics. Motivated by this, we introduce a new geometric flow which is designed to deform a given metric to a dHYM one. Then we show that this new flow potentially has more global existence and convergence properties than the LBMCF.

*Chun-Jun Tsai (National Taiwan University, Taiwan)*

## **Strong stability of minimal submanifolds**

### **Abstract:**

In this talk, we will first explain a notion of stability of minimal submanifolds and its geometric implications. We will then focus on a famous class of hyper-Kähler 4-manifolds, and explain some interesting questions there. Part of this is based on a joint work with Mu-Tao Wang.

*Chin-Tung Wu (National Pingtung University, Taiwan)*

## **Harnack estimates for the Sasaki-Ricci flow**

### **Abstract:**

In this talk, we derive the Li-Yau-Hamilton estimates and the Harnack inequalities for the Sasaki-Ricci flow on a Sasaki  $(2n+1)$ -manifold with nonnegative transverse bisectional curvature. As applications, we show that the blow-down limit of certain type of long-time solution is a transverse Sasaki-Ricci soliton and any initial Sasakian metric on  $S^3$  will converge to the Sasaki-Ricci soliton. This work is joint with S.-C. Chang and Y. Han.