Abstracts

Quasilinear elliptic problems with unbounded measure data

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We discuss the solvability of p-Poisson equations with unbounded measure data. If the boundary of the domain is smooth and p=2, a necessary and sufficient condition for the existence of a solution is that the distance from the boundary is integrable. Similar results are not known for nonlinear equations. We solve this problem using variational methods and comparison principles and investigate sufficient conditions of the existence of solutions. In addition, we discuss the relation between the conditions and Hardy's inequality.

Sobolev-type inequalities in quantitative form

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Broadly speaking, Sobolev-type inequalities relate the integrability or regularity of a function to the integrability of its gradient. In recent years, there has been significant interest in understanding quantitative versions of these inequalities. In these lectures, I will discuss various results in this direction, with a focus toward techniques and proof ideas.

On the Hardy and CKN type inequalities involving non-doubling weights

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We will give lectures on the Hardy and CKN type inequalities with "non-doubling weights" in a domain Ω of $\mathbf{R^n}$. By "non-doubling weights" we mean rather general ones that may vanish or blow up in infinite order such as $e^{-1/t}$ and $e^{1/t}$ at $t = 0$ in one dimensional case. By $W(\mathbf{R}_+)$ we denote a set of all functions $\{w \in C^1(\mathbf{R}_+) : w > 0\}$ with $\mathbf{R}_+ = (0, \infty)$. As classes of weight functions on $\mathbf{R}_+$, we will introduce $P(\mathbf{R}_+)$ and

$Q(\mathbf{R}_+)$, and later $P_A(\mathbf{R}_+)$ and $Q_A(\mathbf{R}_+)$ as subclasses. As weights we adopt $W_p(t) = w(t)^{p-1}$ with $w(t) \in P(\mathbf{R}_+) \cup Q(\mathbf{R}_+)$, where

$$\begin{cases} P(\mathbf{R}_+) = \{w(t) \in W(\mathbf{R}_+) : \, w(t)^{-1} \notin L^1((0,\eta)) \text{ for any } \eta > 0\}, \\ Q(\mathbf{R}_+) = \{w(t) \in W(\mathbf{R}_+) : \, w(t)^{-1} \in L^1((0,\eta)) \text{ for any } \eta > 0\}. \end{cases} \tag{1}$$

Clearly $W(\mathbf{R}_+) = P(\mathbf{R}_+) \cup Q(\mathbf{R}_+)$, $e^{-1/t} \in P(\mathbf{R}_+)$ and $e^{1/t} \in Q(\mathbf{R}_+)$ for $t > 0$.

In the classical inequalities, powers of the distance $\delta(x)$ (typically to the boundary $\partial\Omega$) used to be adopted as weights. But in the present talk we work with (non-doubling) weights such as $W_p(\delta(x)) = (W_p \circ \delta)(x)$ with $W_p(t) = w^{p-1}(t)$ $(w \in W(\mathbf{R}_+))$ in addition to power-type weights $\delta(x)^\alpha$ $(\alpha \in \mathbf{R})$.

We also discuss two applications of our Hardy type inequalities. The one is a variational problem with Hardy potentials in a bounded domain Ω of $\mathbf{R^n}$ and the other is a new class of Caffarelli-Kohn-Nirenberg type inequalities in a ball $B_R(0)$ (R is possibly $+\infty$). The both problems contain interesting open problems at least to the author.

As a compass, we carry simple Hardy's inequalities with non-doubling weights.

One dimensional example:

Assume that $1 < p < \infty$, $\mu > 0$ and $\eta > 0$. $\Lambda_p = (1 - 1/p)^p$

1. For every $u \in C_c^1((0,\eta])$ we have

$$\int_0^\eta |u'(t)|^p e^{-(p-1)/t}\, dt + \frac{(\Lambda_p)^{1/p'}}{\mu^{p-1}} |u(\eta)|^p \geq \Lambda_p \int_0^\eta \frac{|u(t)|^p e^{-(p-1)/t}\, dt}{\left(e^{-1/t}\left(\int_t^\eta e^{1/s}\, ds + \mu\right)\right)^p}.$$

2. For every $u \in C_c^1((0,\eta])$ we have

$$\int_0^\eta |u'(t)|^p e^{(p-1)/t}\, dt \geq \Lambda_p \int_0^\eta \frac{|u(t)|^p e^{(p-1)/t}\, dt}{\left(e^{1/t}\int_0^t e^{-1/s}\, ds\right)^p} + \frac{(\Lambda_p)^{1/p'}}{\left(\int_0^\eta e^{-1/s}\, ds\right)^{p-1}} |u(\eta)|^p.$$