INVARIANT SUBRINGS OF THE COX RING OF K3 SURFACES (ANNOUNCEMENT)

AKIYOSHI SANNAI

Abstract. This is an announcement of [Sa]. We consider the invariant subring of the Cox ring by the automorphism group of the projective variety X. We prove the ring is finitely generated if X is K3 surfaces.

1. result and motivations

Cox rings were introduced by D.Cox in [C] and are important rings which appeared in algebraic geometry.

Definition 1.1 (Multi-section rings and Cox rings)**.** Let X be a normal projective variety over the algebraic closed field k. For a semi-group Γ of Weil divisors on X, the Γ-graded ring

$$R_X(\Gamma) = \bigoplus_{D \in \Gamma} H^0(X, \mathcal{O}_X(D))$$

is called the *multi-section ring* of Γ.

Suppose that $\mathrm{Cl}\,(X)$ is freely finitely generated. For such X, choose a group Γ of Weil divisors on X such that $\Gamma \to \mathrm{Cl}\,(X)$ is an isomorphism. Then the multi-section ring $R_X(\Gamma)$ is called a *Cox ring* of X and it does not depend on the choice of Γ.

Remark 1.2. $Aut(X)$ naturally acts $Cox(X)$ by pulling back the sections. Thus we can consider the invariant subring of $Cox(X)$ by $Aut(X)$.

One of the main topic related with Cox rings is the finite generation of the rings. We consider the Cox ring of K3 surfaces.

Definition 1.3. A K3 surface over k is a smooth projective surface X such that $\omega_{X/k} = \mathcal{O}_X$ and $H^1(X, \mathcal{O}_X) = 0$.

The main theorem is the following:

Date: June 3, 2014.

2010 *Mathematics Subject Classification.* Primary 14J45; Secondary 13A35, 14B05, 14E30.

Key words and phrases. K3surfaces, automorphism, invariant subring, Cox ring.

Theorem 1.4. *Let X be a K3 surface defined over $\mathbb{C}$. Then the invariant subring of $Cox(X)$ by $Aut(X)$ is finitely generated over $\mathbb{C}$.*

The theorem has the following meaning. At first, by [BCHM] and [DAM], we know the Cox rings are finitely generated if the base varieties are of Fano type or smooth surfaces with big anti canonical divisor. This means that the positivity of the anti canonical divisors make the Cox rings finitely generated. Secondly, we consider the case in which the anti canonical divisor is trivial. If the variety is an Abelian variety, then the picard group is not finitely generated. We can not define the Cox ring. Therefore, we consider the K3 surfaces. By [K] and [AHL], we know the Cox ring of a K3 surface is finitely generated if and only if the automorphism group is finite group. Therefore, we have both of the examples of finitely generated and infinitely generated. This implies that if the Cox ring is finitely generated, then the invariant subring of the Cox ring by the auto morphism is finitely generated. It is natural to ask whether this holds in general or not. This question is first asked in "lectures on K3 surfaces" by D. Huybrechts [H].

References

[AHL] M. Artebani, J. Hausen, and A. Laface, On Cox rings of K3 surfaces, Compos. Math. **146** (2010), no. 4, 964-998.

[BCHM] C. Birkar, P. Cascini, C. Hacon, and J. McKernan, Existence of minimal models for varieties of log general type, J. Amer. Math. Soc. 23, no. 2, 405-468.

[C] D. A. Cox, The homogeneous coordinate ring of a toric variety, J. Algebraic Geometry **4** 17-50 (1995), .

[DAM] D. Testa, A. Varilly-Alvarado, and M. Velasco, Big rational surfaces, Math. Ann. **351** (2011), no. 1, 95-107.

[FG] Osamu. F, Yoshinori. G, Log pluricanonical representations and the abundance conjecture, to appear in Compositio Math.

[H] D.Huybrechts, lectures on K3 surfaces, available at http://www.math.uni-bonn.de/people/huybrech/K3Global.pdf

[K] S. Kovacs, The cone of curves of a K3 surface. Math. Ann., **300** (4):681-691, 1994.

[Sa] A. Sannai, Invariant subrings of the Cox ring of K3 surfaces. In preparation.

[St] Sterk. H, Finiteness Results for Algebraic K3 surfaces, Math. Z. **189**, 507-513(1985)

[U] K. Ueno, Classification theory of algebraic varieties and compact complex spaces, Lecture Notes in Math., **439**, Springer, Berlin, 1975.

Graduate School of Mathematical Sciences, University of Tokyo, Meguro, Tokyo, 153-9814, Japan.

E-mail address: sannai@ms.u-tokyo.ac.jp