Errata

Local cohomology on diagrams of schemes

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As declared at the top of section 5 of the above mentioned article, Lemma 5.8 there was taken from Kempf's article [2, Proposition 6]. However, the way of citing was erroneous. In Kempf's original paper, the intersection of two quasi-compact open subsets is assumed to be again quasi-compact. But in Lemma 5.8, this assumption has been erroneously omitted, and as the following example shows, the lemma in the present form is wrong. The correct statement of Lemma 5.8 is given below.

Lemma 5.8 Let X be a topological space. Assume that X has an open basis consisting of quasi-compact open subsets, and that the intersection of two quasi-compact open subsets is quasi-compact. Let U be a quasi-compact open subset of X, and $(\mathcal{M}_\lambda)$ a pseudo-filtered inductive system of sheaves of abelian groups on X. Then the canonical map

$$\varinjlim \Gamma(U, \mathcal{M}_\lambda) \to \Gamma(U, \varinjlim \mathcal{M}_\lambda)$$

is an isomorphism.

In the statement of Corollary 5.10 we need to assume that X is quasi-separated and in its proof U should be assumed to be affine.

In Lemma 5.11, we use Corollary 5.10, but since an immersion is quasi-separated, the lemma requires no modification.

Example 1. Let $P = \mathbb{N} \cup \{a, b\}$ be a partially ordered set defined by:

(1) any two elements of $\mathbb{N}$ are incomparable;

(2) a and b are incomparable;

(3) any element of $\{a, b\}$ is smaller than each element of $\mathbb{N}$.

Letting a filter (or an increasing subset) an open subset, P is a topological space. For $x \in P$, let $V_x := \{y \in P \mid y \geq x\}$ be the principal filter generated by x. Note that V_x is quasi-compact, and $\{V_x \mid x \in P\}$ is an open basis of P. Note also that $P = V_a \cup V_b$ is also quasi-compact. However, the intersection $V_a \cap V_b$ of the two quasi-compact open subsets V_a and V_b is $\mathbb{N}$, which is not quasi-compact.

Let k be a field. For $n \in \mathbb{N}$, define a sheaf M_n on P by

(1) $\Gamma(V_m, M_n) = k$ if $m \geq n$, and $\Gamma(V_m, M_n) = 0$ if $m < n$.

(2) $\Gamma(V_a, M_n) = \Gamma(V_b, M_n) = k$;

(3) The restriction $\Gamma(V_c, M_n) \to \Gamma(V_m, M_n)$ is the identity map of k for $c \in \{a, b\}$ and $m \geq n$.

This uniquely determines a sheaf M_n of k-vector spaces for $n \in \mathbb{N}$, see [1]. For $n' \geq n$, we define $\varphi_{n',n} : M_n \to M_{n'}$ by

(1) $\Gamma(V_m, \varphi_{n',n})$ is the identity of k if $m \geq n'$.

(2) $\Gamma(V_a, \varphi_{n',n})$ and $\Gamma(V_b, \varphi_{n',n})$ are the identity of k.

Note that $\Gamma(V_m, \varinjlim M_n) = \varinjlim \Gamma(V_m, M_n) = 0$. So $\Gamma(\mathbb{N}, \varinjlim M_n) = \prod_{m \in \mathbb{N}} \Gamma(V_m, \varinjlim M_n) = 0$. By the exact sequence

$$0 \to \Gamma(P, \varinjlim M_n) \to \Gamma(V_a, \varinjlim M_n) \oplus \Gamma(V_b, \varinjlim M_n) \to \Gamma(\mathbb{N}, \varinjlim M_n) = 0,$$

$\Gamma(P, \varinjlim M_n)$ is two dimensional. On the other hand, $\Gamma(P, M_n)$ is always the diagonal subgroup of $k \oplus k = \Gamma(V_a, M_n) \oplus \Gamma(V_b, M_n)$, and hence $\varinjlim \Gamma(P, M_n)$ is one dimensional.

This shows $\varinjlim \Gamma(P, M_n) \not\cong \Gamma(P, \varinjlim M_n)$, although P is quasi-compact with an open basis consisting of quasi-compact open subsets. □

After Lemma 2.5, $f^{\#}\mathcal{O}_{\mathbb{Y}}$ should be $f_{\#}\mathcal{O}_{\mathbb{Y}}$.

In Lemma 5.13, $\mathcal{M}$ is a sheaf of abelian groups on X such that $\mathcal{M}_i$ is quasi-flabby for each $i \in I$. In the proof, the problem is reduced to the single-scheme case.

In the proof of Corollary 6.3, $\bigcup_\lambda W_\lambda$ should be $(W_\lambda)_\lambda$.

References

[1] K. Baclawski, Whitney numbers of geometric lattices, *Adv. Math.* **16** (1975), 125–138.

[2] G. R. Kempf, Some elementary proofs of basic theorems in the cohomology of quasi-coherent sheaves, *Rocky Mountain J. Math.* **10** (1980), 637–645.