F-finiteness of homomorphisms and its descent

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Abstract

Let p be a prime number. We define the notion of F-finiteness of homomorphisms of $\mathbb{F}_p$-algebras, and discuss some basic properties. In particular, we prove a sort of descent theorem on F-finiteness of homomorphisms of $\mathbb{F}_p$-algebras. As a corollary, we prove the following. Let $g : B \to C$ be a homomorphism of Noetherian $\mathbb{F}_p$-algebras. If g is faithfully flat reduced and C is F-finite, then B is F-finite. This is a generalization of Seydi's result on excellent local rings of characteristic p.

1. Introduction

Throughout this paper, p denotes a prime number, and $\mathbb{F}_p$ denotes the finite field with p elements.

The notions of Nagata (pseudo-geometric, universally Japanese) and (quasi-)excellent rings give good frameworks to avoid pathologies which appear in the theory of Noetherian rings, see [Nag], [Gro], and [Mat].

In commutative algebra of characteristic p, F-finiteness of rings is commonly used for a general assumption which guarantees the "tameness" of

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the theory, as well as Nagata and (quasi-)excellent properties. A commutative ring R of characteristic p is said to be F-finite if the Frobenius map $F_R : R \to R$ ($F_R(r) = r^p$) is finite (that is, R as the target of F_R is a finite module over R as the source of F_R). As the definition suggests, F-finiteness is important in studying ring theoretic properties defined via Frobenius maps, such as strong F-regularity [HH]. Although F-finiteness for a Noetherian $\mathbb{F}_p$-algebra is stronger than excellence [Kun], F-finiteness is not so restrictive for practical use. A perfect field is F-finite. An algebra essentially of finite type over an F-finite ring is F-finite. An ideal-adic completion of a Noetherian F-finite ring is again F-finite. See Example 3 and Example 9. It is known that an F-finite Noetherian ring is a homomorphic image of an F-finite regular ring of finite Krull dimension, and hence it has a dualizing complex [Gab, Remark 13.6].

In this paper, replacing the absolute Frobenius map by the relative one, we define the F-finiteness of homomorphism between rings of characteristic p. We say that an $\mathbb{F}_p$-algebra map $A \to B$ is F-finite (or B is F-finite over A) if the relative Frobenius map (Radu–André homomorphism) $\Phi_1(A, B) : B^{(1)} \otimes_{A^{(1)}} A \to B$ is finite (Definition 1, see section 2 for the notation). Thus a ring B of characteristic p is F-finite if and only if it is F-finite over $\mathbb{F}_p$. Replacing absolute Frobenius by relative Frobenius, we get definitions and results on homomorphisms instead of rings. This is a common idea in [Rad], [And2], [And3], [Dum], [Dum2], [Ene], [Has], [DI], and [Has2].

In section 2, we discuss basic properties of F-finiteness of homomorphisms and rings. Some of well-known properties of F-finiteness of rings are naturally generalized to those for F-finiteness of homomorphisms. F-finiteness of homomorphisms has connections with that for rings. For example, if $A \to B$ is F-finite and A is F-finite, then B is F-finite (Lemma 2).

In section 3, we prove the main theorem (Theorem 21). This is a sort of descent of F-finiteness. As a corollary, we prove that for a faithfully flat reduced homomorphism of Noetherian rings $g : B \to C$, if C is F-finite, then B is F-finite. Considering the case that f is a completion of a Noetherian local ring, we recover Seydi's result on excellent local rings of characteristic p [Sey].

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2. F-finiteness of homomorphisms

Let k be a perfect field of characteristic p, and $r \in \mathbb{Z}$. For a k-space V, the additive group V with the new k-space structure $\alpha \cdot v = \alpha^{p^{-r}} v$ is denoted by $V^{(r)}$. An element v of V, viewed as an element of $V^{(r)}$ is (sometimes) denoted by $v^{(r)}$. If A is a k-algebra, then $A^{(r)}$ is a k-algebra with the product $a^{(r)} \cdot b^{(r)} = (ab)^{(r)}$. We denote the Frobenius map $A \to A$ ($a \mapsto a^p$) by F or F_A. Note that $F^e : A^{(r+e)} \to A^{(r)}$ is a k-algebra map. Throughout the article, we regard $A^{(r)}$ as an $A^{(r+e)}$-algebra through F^e (A is viewed as $A^{(0)}$). For an A-module M, the action $a^{(r)} \cdot m^{(r)} = (am)^{(r)}$ makes $M^{(r)}$ an $A^{(r)}$-module. If I is an ideal of A, then $I^{(r)}$ is an ideal of $A^{(r)}$. If $e \geq 0$, then $I^{(e)} A = I^{[p^e]}$, where $I^{[p^e]}$ is the ideal of A generated by $\{a^{p^e} \mid a \in I\}$. In commutative algebra, $A^{(r)}$ is also denoted by ${}^{-r}A$. We employ the notation more consistent with that in representation theory — the eth Frobenius twist of V is denoted by $V^{(e)}$, see [Jan]. We use this notation for $k = \mathbb{F}_p$.

Let $A \to B$ be an $\mathbb{F}_p$-algebra map, and $e \geq 0$. Then the relative Frobenius map (or Radu–André homomorphism) $\Phi_e(A, B) : B^{(e)} \otimes_{A^{(e)}} A \to B$ is defined by $\Phi_e(A, B)(b^{(e)} \otimes a) = b^{p^e} a$.

Definition 1. An $\mathbb{F}_p$-algebra map $A \to B$ is said to be *F-finite* if $\Phi_1(A, B) : B^{(1)} \otimes_{A^{(1)}} A \to B$ is finite. That is, B is a finitely generated $B^{(1)} \otimes_{A^{(1)}} A$-module through $\Phi_1(A, B)$. We also say that B is F-finite over A.

Lemma 2. *Let $f : A \to B$, $g : B \to C$, and $h : A \to \tilde{A}$ be $\mathbb{F}_p$-algebra maps, and $\tilde{B} := \tilde{A} \otimes_A B$.*

1 *The following are equivalent.*

- **a** *f is F-finite. That is, $\Phi_1(A, B)$ is finite.*
- **b** *For any $e > 0$, $\Phi_e(A, B)$ is finite.*
- **c** *For some $e > 0$, $\Phi_e(A, B)$ is finite.*

2 *If f and g are F-finite, then so is gf.*

3 *If gf is F-finite, then so is g.*

4 *The ring A is F-finite (that is, the Frobenius map $F_A : A^{(1)} \to A$ is finite) if and only if the unique homomorphism $\mathbb{F}_p \to A$ is F-finite.*

5 *If $f : A \to B$ is F-finite, then the base change $\tilde{f} : \tilde{A} \to \tilde{B}$ is F-finite.*

6 *If B is F-finite, then f is F-finite.*

7 *If A and f are F-finite, then B is F-finite.*

Proof. **1** This is immediate, using [Has, Lemma 4.1, **2**]. **2** and **3** follow from [Has, Lemma 4.1, **1**]. **4** follows from [Has, Lemma 4.1, **5**]. **5** follows from [Has, Lemma 4.1, **4**]. **6** follows from **3** and **4**. **7** follows from **2** and **4**. $\square$

Example 3. Let $e \geq 1$, and $f : A \to B$ be an $\mathbb{F}_p$-algebra map.

1 If $B = A[x]$ is a polynomial ring, then it is F-finite over A.

2 If $B = A_S$ is a localization of A by a multiplicatively closed subset S of A, then $\Phi_e(A, B)$ is an isomorphism. In particular, B is F-finite over A.

3 If $B = A/I$ with I an ideal of A, then

$$B^{(e)} \otimes_{A^{(e)}} A \cong (A^{(e)}/I^{(e)}) \otimes_{A^{(e)}} A \cong A/I^{(e)}A = A/I^{[p^e]}.$$

Under this identification, $\Phi_e(A, B)$ is identified with the projection $A/I^{[p^e]} \to A/I$. In particular, B is F-finite over A.

4 If B is essentially of finite type over A, then B is F-finite over A.

Proof. **1** The image of $\Phi_1(A, B)$ is $A[x^p]$, and hence B is generated by $1, x, \ldots, x^{p-1}$ over it. **2** Note that $B^{(e)}$ is identified with $(A^{(e)})_{S^{(e)}}$, where $S^{(e)} = \{s^{(e)} \mid s \in S\}$. So $B^{(e)} \otimes_{A^{(e)}} A$ is identified with $(A^{(e)})_{S^{(e)}} \otimes_{A^{(e)}} A \cong A_{S^{(e)}}$, and $\Phi_e(A, B)$ is identified with the isomorphism $A_{S^{(e)}} \cong A_S$. **3** is obvious. **4** This is a consequence of **1**, **2**, **3**, and Lemma 2, **2**. $\square$

Lemma 4. *Let $A \xrightarrow{f} B \xrightarrow{g} C$ be a sequence of $\mathbb{F}_p$-algebra maps. Then for $e > 0$, the diagram*

$$\begin{array}{ccc} B^{(e)} \otimes_{A^{(e)}} A & \xrightarrow{\Phi_e(A,B)} & B \\ \downarrow{\scriptstyle g^{(e)} \otimes 1} & & \downarrow{\scriptstyle g} \\ C^{(e)} \otimes_{A^{(e)}} A & \xrightarrow{\Phi_e(A,C)} & C \end{array}$$

is commutative.

Proof. This is straightforward. $\square$

Lemma 5. *Let $A \xrightarrow{f} B \xrightarrow{g} C$ be a sequence of $\mathbb{F}_p$-algebra maps, and assume that C is F-finite over A. If g is finite and injective, and $B^{(e)} \otimes_{A^{(e)}} A$ is Noetherian for some $e > 0$, then B is F-finite over A.*

Proof. By assumption, $C^{(e)} \otimes_{A^{(e)}} A$ is finite over $B^{(e)} \otimes_{A^{(e)}} A$, and C is finite over $C^{(e)} \otimes_{A^{(e)}} A$. So C is finite over $B^{(e)} \otimes_{A^{(e)}} A$. As B is a $B^{(e)} \otimes_{A^{(e)}} A$-submodule of C and $B^{(e)} \otimes_{A^{(e)}} A$ is Noetherian, B is finite over $B^{(e)} \otimes_{A^{(e)}} A$. □

Lemma 6. *Let $A \to B$ be a ring homomorphism, and I a finitely generated nilpotent ideal of B. If B/I is A-finite, then B is A-finite.*

Proof. As I^i/I^{i+1} is B/I-finite for each i, it is also A-finite. So B/I^r is A-finite for each r. Taking r large, B is A-finite. □

Lemma 7. *Let $f : A \to B$ be an $\mathbb{F}_p$-algebra map, and I a finitely generated nilpotent ideal of B. If B/I is F-finite over A, then B is F-finite over A.*

Proof. As B/I is F-finite over A, B/I is $(B^{(1)}/I^{(1)}) \otimes_{A^{(1)}} A$-finite. So B/I is also $B^{(1)} \otimes_{A^{(1)}} A$-finite. By Lemma 6, B is $B^{(1)} \otimes_{A^{(1)}} A$-finite. □

For the absolute F-finiteness, we have a better result.

Lemma 8. *Let B be an $\mathbb{F}_p$-algebra, and I a finitely generated ideal of B. If B is I-adically complete and B/I is F-finite, then B is F-finite.*

Proof. B/I is $B^{(1)}/I^{(1)}$-finite. So $B/I^{(1)}B$ is $B^{(1)}$-finite by Lemma 6. As $\bigcap_i I^i = 0$, we have $\bigcap_i (I^{(1)})^i B = 0$. Moreover, $B^{(1)}$ is $I^{(1)}$-adically complete. Hence B is $B^{(1)}$-finite by [Mat2, Theorem 8.4]. □

Example 9. Let A be an $\mathbb{F}_p$-algebra.

1 If A is F-finite, then the formal power series ring $A[[x]]$ is so.

2 Let J be an ideal of A. If A is Noetherian and A/J is F-finite, then the J-adic completion A^* of A is F-finite.

3 If $(A, \mathfrak{m})$ is complete local and $A/\mathfrak{m}$ is F-finite, then A is F-finite.

Proof. For each of **1–3**, we use Lemma 8. **1** Set $B = A[[x]]$ and $I = Bx$. Then $B/I \cong A$ is F-finite. **2** Set $B = A^*$ and $I = JB$. Then $B/I \cong A/J$ is F-finite. **3** is immediate. □

Remark 10. Let A be a Noetherian ring and I its ideal. If A is I-adically complete and A/I is Nagata, then A is Nagata [Mar]. If A is semi-local, I-adically complete, and A/I is quasi-excellent, then A is quasi-excellent [Rot2]. See also [Nis].

Lemma 11. *Let A be an $\mathbb{F}_p$-algebra, and B and C be A-algebras. If B and C are F-finite over A, then*

1 *$B \otimes_A C$ is F-finite over A.*

2 *$B \times C$ is F-finite over A.*

Proof. **1** B is F-finite over A, and $B \otimes_A C$ is F-finite over B by Lemma 2, **5**. By Lemma 2, **2**, $B \otimes_A C$ is F-finite over A.

2 Both B and C are finite over $(B \times C)^{(1)} \otimes_{A^{(1)}} A$, and so is $B \times C$. □

Lemma 12. *Let $A \to B$ be an $\mathbb{F}_p$-algebra map, and assume that B and $B^{(e)} \otimes_{A^{(e)}} A$ are Noetherian for some $e > 0$. Then B is F-finite over A if and only if B/P is F-finite over A for every minimal prime P of B.*

Proof. The 'only if' part is obvious by Example 3, **3**. We prove the converse. Let $\operatorname{Min} B$ be the set of minimal primes of B. Then $\prod_{P \in \operatorname{Min} B} B/P$ is F-finite over A by Lemma 11. As $B_{\mathrm{red}} \to \prod_{P \in \operatorname{Min} B} B/P$ is finite injective, and $B_{\mathrm{red}}^{(e)} \otimes_{A^{(e)}} A$ is Noetherian, B_{red} is F-finite over A by Lemma 5. As B is Noetherian, B is F-finite over A by Lemma 7. □

Remark 13. Fogarty asserted that an $\mathbb{F}_p$-algebra map $A \to B$ with B Noetherian is F-finite if and only if the module of Kähler differentials $\Omega_{B/A}$ is a finite B-module [Fog, Proposition 1]. The 'only if' part is true and easy. The proof of 'if' part therein has a gap. Although R_1 in step (iii) is assumed to be Noetherian, it is not proved that R' in step (iv) is Noetherian. The author does not know if this direction is true or not.

If, moreover, both A and B are Noetherian, the assertion is true. This is an immediate consequence of [And, Proposition 57].

3. Descent of F-finiteness

In this section, we prove a sort of descent theorem on F-finiteness of homomorphisms.

Let R be a commutative ring, and $f : M \to N$ an R-linear map between R-modules. We say that f is pure, if $1_W \otimes f : W \otimes_R M \to W \otimes_R N$ is injective for any R-module W. When we need to clarify the base ring R, we also say that f is R-pure. A homomorphism of rings $A \to B$ is said to be pure (without mentioning the base ring), if it is A-pure (i.e., pure as an A-linear map).

Lemma 14. *Let R be a commutative ring, $\varphi : M \to N$ and $h : F \to G$ be R-linear maps. If φ is R-pure and $1_N \otimes h : N \otimes F \to N \otimes G$ is surjective, then $1_M \otimes h : M \otimes F \to M \otimes G$ is surjective.*

Proof. Let $C := \operatorname{Coker} h$. Then by assumption, $N \otimes C = 0$. By the injectivity of $\varphi \otimes 1_C : M \otimes C \to N \otimes C$, we have that $M \otimes C = 0$. □

Corollary 15. *Let $A \to B$ be a pure ring homomorphism, and $h : F \to G$ an A-linear map. If $1_B \otimes h : B \otimes_A F \to B \otimes_A G$ is surjective, then h is surjective.* □

Lemma 16. *Let $A \to B$ be a pure ring homomorphism, and G an A-module. If $B \otimes_A G$ is a finitely generated B-module, then G is finitely generated as an A-module.*

Proof. Let $\theta_1, \ldots, \theta_r$ be generators of $B \otimes_A G$. Then we can write $\theta_j = \sum_{i=1}^{s} b_{ij} \otimes g_{ij}$ for some $s > 0$, $b_{ij} \in B$, and $g_{ij} \in G$. Let F be the A-free module with the basis $\{f_{ij} \mid 1 \leq i \leq s,\ 1 \leq j \leq r\}$, and $h : F \to G$ be the A-linear map given by $f_{ij} \mapsto g_{ij}$. Then by construction, $1_B \otimes h$ is surjective. By Corollary 15, h is surjective, and hence G is finitely generated. □

Definition 17 (cf. [Has2, (2.7)]). Let $e > 0$ be an integer. An $\mathbb{F}_p$-algebra map $A \to B$ is said to be e-Dumitrescu if $\Phi_e(A, B)$ is A-pure.

Lemma 18. *Let $e, e' > 0$. If $A \to B$ is both e-Dumitrescu and e'-Dumitrescu, then it is $(e + e')$-Dumitrescu. In particular, an e-Dumitrescu map is er-Dumitrescu map for $r > 0$.*

Proof. This follows from [Has, Lemma 4.1, **2**]. □

So a 1-Dumitrescu map is Dumitrescu (that is, e-Dumitrescu for all $e > 0$), see [Has2, Lemma 2.9].

Lemma 19. *Let $e > 0$.*

1 [Has2, Lemma 2.8],

2 [Has2, Lemma 2.12], *and*

3 [Has2, Corollary 2.13]

hold true when we replace all the 'Dumitrescu' therein by 'e-Dumitrescu'.

The proof is straightforward, and is left to the reader.

Remark 20. The precise statement of Lemma 19 for **2** is as follows.

Let $f : A \to B$ be a ring homomorphism between rings of characteristic p, and $e > 0$ an integer. Assume that A is Noetherian, and the image of the associated map ${}^a f : \operatorname{Spec} B \to \operatorname{Spec} A$ contains $\operatorname{Max}(A)$, the set of maximal ideals of A. If f is e-Dumitrescu, then f is pure.

Theorem 21. *Let $f : A \to B$ and $g : B \to C$ be $\mathbb{F}_p$-algebra maps, and $e > 0$. Assume that g is e-Dumitrescu, and the image of the associated map ${}^a g : \operatorname{Spec} C \to \operatorname{Spec} B$ contains the set of maximal ideals $\operatorname{Max} B$ of B. If gf is F-finite, and B and $C^{(e)} \otimes_{A^{(e)}} A$ are Noetherian, then f is F-finite.*

Proof. Note that $\Phi_e(A, C) : C^{(e)} \otimes_{A^{(e)}} A \to C$ is a finite map. Note also that $C^{(e)} \otimes_{B^{(e)}} B$ is a $C^{(e)} \otimes_{A^{(e)}} A$-submodule of C through $\Phi_e(B, C)$, since $\Phi_e(B, C)$ is B-pure and hence is injective. As $C^{(e)} \otimes_{A^{(e)}} A$ is Noetherian, $C^{(e)} \otimes_{B^{(e)}} B$, which is a submodule of the finite module C, is a finite $C^{(e)} \otimes_{A^{(e)}} A$-module. Since $g^{(e)} : B^{(e)} \to C^{(e)}$ is pure by Lemma 19, **2** (see Remark 20), $B^{(e)} \otimes_{A^{(e)}} A \to C^{(e)} \otimes_{A^{(e)}} A$ is also pure. Since

$$C^{(e)} \otimes_{B^{(e)}} B \cong (C^{(e)} \otimes_{A^{(e)}} A) \otimes_{B^{(e)} \otimes_{A^{(e)}} A} B$$

is a finite $C^{(e)} \otimes_{A^{(e)}} A$-module, B is a finite $B^{(e)} \otimes_{A^{(e)}} A$-module by Lemma 16. □

A homomorphism $f : A \to B$ between Noetherian rings is said to be reduced if f is flat with geometrically reduced fibers.

Corollary 22. *Let $g : B \to C$ be a faithfully flat reduced homomorphism between Noetherian $\mathbb{F}_p$-algebras. If C is F-finite, then B is F-finite.*

Proof. By [Dum2, Theorem 3], g is Dumitrescu. As g is faithfully flat, ${}^a g : \operatorname{Spec} C \to \operatorname{Spec} B$ is surjective. Letting $A = \mathbb{F}_p$ and $f : A \to B$ be the unique map, the assumptions of Theorem 21 are satisfied, and hence f is F-finite. That is, B is F-finite. □

Corollary 23 (Seydi [Sey])**.** *Let* $(B, \mathfrak{m})$ *be a Nagata local ring with the* F*-finite residue field* $k = B/\mathfrak{m}$*. Then* B *is* F*-finite. In particular,* B *is excellent, and is a homomorphic image of an* F*-finite regular local ring. So* B *has a dualizing complex.*

Proof. Let $g : B \to C = \hat{B}$ be the completion of B. Then C is a complete local ring with the residue field k. By Example 9, **3**, C is F-finite. As g is reduced by [Gro, (7.6.4), (7.7.2)], B is F-finite by Corollary 22.

Now B is excellent by [Kun, Theorem 2.5] and is a homomorphic image of an F-finite regular local ring by [Gab, Remark 13.6]. The last assertion follows from the fact that a homomorphic image of a Gorenstein ring has a dualizing complex if it is of finite Krull dimension. For dualizing complexes, see [Har]. □

Even if $A \to B$ is a faithfully flat reduced homomorphism and B is excellent, A need not be quasi-excellent. There is a Nagata local ring A which is not quasi-excellent [Rot], [Nis2], and its completion $A \to \hat{A} = B$ is an example. On the other hand, if $A \to B$ is a faithfully flat regular homomorphism and B is quasi-excellent, then A is quasi-excellent [Mat2, Theorem 32.2].

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